Quantum Field Theory

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*"Relativistic quantum field theory" redirects here. For other uses, see Relativity.* In theoretical physics, quantum field theory (QFT) is the theoretical framework for constructing quantum mechanical models of subatomic particles in particle physics and quasiparticles in condensed matter physics. QFT treats particles as excited states of the underlying physical field, so these are called field quanta.

In quantum field theory, quantum mechanical interactions among particles are described by interaction terms among the corresponding underlying quantum fields. These interactions are conveniently visualized by Feynman diagrams, which are a formal tool of relativistically covariant perturbation theory, serving to evaluate particle processes.

History

*Main article: History of quantum field theory*

Even though QFT is an unavoidable consequence of the reconciliation of quantum mechanics with special relativity (Weinberg (2005)), historically, it emerged in the 1920s with the quantization of the electromagnetic field (the quantization being based on an analogy of the eigenmode expansion of a vibrating string with fixed endpoints).

**Early development**

Max Born (1882–1970), one of the founders of quantum field theory.

He is also known for the Born rule that introduced the probabilistic interpretation in quantum mechanics. He received the 1954 Nobel Prize in Physics together with Walther Bothe.

The first achievement of quantum field theory, namely quantum electrodynamics (QED), is "still the paradigmatic example of a successful quantum field theory" ( Weinberg (2005)). Ordinarily, quantum mechanics (QM) cannot give an account of photons which constitute the prime case of relativistic 'particles'. Since photons have rest mass zero, and correspondingly travel in the vacuum at the speed *c*, a non-relativistic theory such as ordinary QM cannot give even an approximate description. Photons are implicit in the emission and absorption processes which have to be postulated; for instance, when one of an atom's electrons makes a transition between energy levels. The formalism of QFT is needed for an explicit description of photons. In fact most topics in the early development of quantum theory (the so-called old quantum theory, 1900–25) were related to the interaction of radiation and matter and thus should be treated by quantum field theoretical methods. However, quantum mechanics as formulated by Dirac, Heisenberg, and Schrödinger in 1926–27 started from atomic spectra and did not focus much on problems of radiation.

As soon as the conceptual framework of quantum mechanics was developed, a small group of theoreticians tried to extend quantum methods to electromagnetic fields. A good example is the famous paper by Born, Jordan & Heisenberg (1926). (P. Jordan was especially acquainted with the literature on light quanta and made seminal contributions to QFT.) The basic idea was that in QFT the electromagnetic field should be represented by matrices in the same way that position and momentum were represented in QM by matrices (matrix mechanics oscillator operators). The ideas of QM were thus extended to systems having an infinite number of degrees of freedom, so an infinite array of quantum oscillators.

The inception of QFT is usually considered to be Dirac's famous 1927 paper on "The quantum theory of the emission and absorption of radiation". Here Dirac coined the name "quantum electrodynamics" (QED) for the part of QFT that was developed first. Dirac supplied a systematic procedure for transferring the characteristic quantum phenomenon of discreteness of physical quantities from the quantum-mechanical treatment of particles to a corresponding treatment of fields. Employing the theory of the quantum harmonic oscillator, Dirac gave a theoretical description of how photons appear in the quantization of the electromagnetic radiation field. Later, Dirac's procedure became a model for the quantization of other fields as well. These first approaches to QFT were further developed during the following three years. P. Jordan introduced creation and annihilation operators for fields obeying Fermi–Dirac statistics. These differ from the corresponding operators for Bose–Einstein statistics in that the former satisfy*anti-commutation relations* while the latter satisfy commutation relations.

The methods of QFT could be applied to derive equations resulting from the quantum-mechanical (field-like) treatment of particles, e.g. the Dirac equation, the Klein–Gordon equation and the Maxwell equations. Schweber points out that the idea and procedure of second quantization goes back to Jordan, in a number of papers from 1927, while the expression itself was coined by Dirac. Some difficult problems concerning commutation relations, statistics, and Lorentz invariance were eventually solved. The first comprehensive account of a general theory of quantum fields, in particular, the method of canonical quantization, was presented by Heisenberg & Pauli in 1929. Whereas Jordan's second quantization procedure applied to the coefficients of the normal modes of the field, Heisenberg & Pauli started with the fields themselves and subjected them to the canonical procedure. Heisenberg and Pauli thus established the basic structure of QFT as presented in modern introductions to QFT. Fermi and Dirac, as well as Fock and Podolsky, presented different formulations which played a heuristic role in the following years.

Quantum electrodynamics rests on two pillars, see e.g., the short and lucid "Historical Introduction" of Scharf (2014). The first pillar is the quantization of the electromagnetic field, i.e., it is about photons as the quantized excitations or 'quanta' of the electromagnetic field. This procedure will be described in some more detail in the section on the particle interpretation. As Weinberg points out the "photon is the only particle that was known as a field before it was detected as a particle" so that it is natural that QED began with the analysis of the radiation field. The second pillar of QED consists of the relativistic theory of the electron, centered on the Dirac equation.

**The problem of infinities**

The emergence of infinities

Pascual Jordan (1902–1980), doctoral student of Max Born, was a pioneer in quantum field theory, coauthoring a number of seminal papers with Born and Heisenberg.

Jordan algebras were introduced by him to formalize the notion of an algebra of observables in quantum mechanics. He was awarded the Max Planck medal 1954.

Quantum field theory started with a theoretical framework that was built in analogy to quantum mechanics. Although there was no unique and fully developed theory, quantum field theoretical tools could be applied to concrete processes. Examples are the scattering of radiation by free electrons, Compton scattering, the collision between relativistic electrons or the production of electron-positron pairs by photons. Calculations to the first order of approximation were quite successful, but most people working in the field thought that QFT still had to undergo a major change. On the one side, some calculations of effects for cosmic rays clearly differed from measurements. On the other side and, from a theoretical point of view more threatening, calculations of higher orders of the perturbation series led to infinite results. The self-energy of the electron as well as vacuum fluctuations of the electromagnetic field seemed to be infinite. The perturbation expansions did not converge to a finite sum and even most individual terms were divergent.

The various forms of infinities suggested that the divergences were more than failures of specific calculations. Many physicists tried to avoid the divergences by formal tricks (truncating the integrals at some value of momentum, or even ignoring infinite terms) but such rules were not reliable, violated the requirements of relativity and were not considered as satisfactory. Others came up with the first ideas for coping with infinities by a redefinition of the parameters of the theory and using a measured finite value, for example of the charge of the electron, instead of the infinite 'bare' value. This process is called renormalization.

From the point of view of the philosophy of science, it is remarkable that these divergences did not give enough reason to discard the theory. The years from 1930 to the beginning of World War II were characterized by a variety of attitudes towards QFT. Some physicists tried to circumvent the infinities by more-or-less arbitrary prescriptions, others worked on transformations and improvements of the theoretical framework. Most of the theoreticians believed that QED would break down at high energies. There was also a considerable number of proposals in favor of alternative approaches. These proposals included changes in the basic concepts e.g. negative probabilities and interactions at a distance instead of a field theoretical approach, and a methodological change to phenomenological methods that focusses on relations between observable quantities without an analysis of the microphysical details of the interaction, the so-called S-matrix theory where the basic elements are amplitudes for various scattering processes.

Despite the feeling that QFT was imperfect and lacking rigor, its methods were extended to new areas of applications. In 1933 Fermi's theory of the beta decay started with conceptions describing the emission and absorption of photons, transferred them to beta radiation and analyzed the creation and annihilation of electrons and neutrinos described by the weak interaction. Further applications of QFT outside of quantum electrodynamics succeeded in nuclear physics with the strong interaction. In 1934 Pauli &Weisskopf showed that a new type of field (the scalar field), described by the Klein–Gordon equation, could be quantized. This is another example of second quantization. This new theory for matter fields could be applied a decade later when new particles, pions, were detected.

**The taming of infinities**

Werner Heisenberg (1901–1976), doctoral student of Arnold Sommerfeld, was one of the founding fathers of quantum mechanics and QFT.

In particular, he introduced the version of quantum mechanics known asmatrix mechanics, but is now more known for the Heisenberg uncertainty relations. He was awarded the Nobel prize in physics 1932.

After the end of World War II more reliable and effective methods for dealing with infinities in QFT were developed, namely coherent and systematic rules for performing relativistic field theoretical calculations, and a general renormalization theory. On three famous conferences, the Shelter Island Conference 1947, the Pocono Conference 1948, and the 1949 Oldstone Conference, developments in theoretical physics were confronted with relevant new experimental results. In the late forties, there were two different ways to address the problem of divergences. One of these was discovered by Richard Feynman, the other one (based on an operator formalism) by Julian Schwinger and, independently, by Sin-Itiro Tomonaga.

In 1949, Freeman Dyson showed that the two approaches are in fact equivalent and fit into an elegant field-theoretic framework. Thus, Freeman Dyson, Feynman, Schwinger, and Tomonaga became the inventors of renormalization theory. The most spectacular successes of renormalization theory were the calculations of the anomalous magnetic moment of the electron and the Lamb shift in the spectrum of hydrogen. These successes were so outstanding because the theoretical results were in better agreement with high-precision experiments than anything in physics encountered before. Nevertheless, mathematical problems lingered on and prompted a search for rigorous formulations (discussed below).

The rationale behind renormalization is to avoid divergences that appear in physical predictions by shifting them into a part of the theory where they do not influence empirical statements. Dyson could show that a rescaling of charge and mass ('renormalization') is sufficient to remove all divergences in QED consistently, to all orders of perturbation theory. A QFT is called renormalizable if all infinities can be absorbed into a redefinition of a *finite number* of coupling constants and masses. A consequence for QED is that the physical charge and mass of the electron must be measured and cannot be computed from first principles.

Perturbation theory yields well-defined predictions only in renormalizable quantum field theories; luckily, QED, the first fully developed QFT, belonged to this class of renormalizable theories. There are various technical procedures to renormalize a theory. One way is to cut off the integrals in the calculations at a certain value *Λ* of the momentum which is large but finite. This cut-off procedure is successful if, after taking the limit *Λ* → ∞, the resulting quantities are independent of *Λ*.

Richard Feynman (1918–1988)
His 1945 PhD thesis developed the path integral formulation of ordinary quantum mechanics. This was later generalized to field theory.

Feynman's formulation of QED is of special interest from a philosophical point of view. His so-called space-time approach is visualized by the celebrated Feynman diagrams that look like depicting paths of particles. Feynman's method of calculating scattering amplitudes is based on the functional integral formulation of field theory. A set of graphical rules can be derived so that the probability of a specific scattering process can be calculated by drawing a diagram of that process and then using that diagram to write down the precise mathematical expressions for calculating its amplitude in relativistically covariant perturbation theory.

The diagrams provide an effective way to organize and visualize the various terms in the perturbation series, and they naturally account for the flow of electrons and photons during the scattering process. External lines in the diagrams represent incoming and outgoing particles, internal lines are connected with virtual particles and vertices with interactions. Each of these graphical elements is associated with mathematical expressions that contribute to the amplitude of the respective process. The diagrams are part of Feynman's very efficient and elegant algorithm for computing the probability of scattering processes.

The idea of particles traveling from one point to another was heuristically useful in constructing the theory. This heuristics, based on Huygen's principle, is useful for concrete calculations and actually give the correct particle propagators as derived more rigorously. Nevertheless, an analysis of the theoretical justification of the space-time approach shows that its success does not imply that particle paths need be taken seriously. General arguments against a particle interpretation of QFT clearly exclude that the diagrams represent actual paths of particles in the interaction area. Feynman himself was not particularly interested in ontological questions.

**The golden age: Gauge theory and the standard model**

Chen-Ning Yang (b.1922), co-inventor of nonabelian gauge field theories.

Murray Gell-Mann (b. 1929) articulator and pioneer of group symmetry in QFT

In 1933, Enrico Fermi had already established that the creation, annihilation and transmutation of particles in the weak interaction beta decay could best be described in QFT, specifically his quartic fermion interaction. As a result, field theory had become a prospective tool for other particle interactions. In the beginning of the 1950s, QED had become a reliable theory which no longer counted as preliminary. However, it took two decades from writing down the first equations until QFT could be applied successfully to important physical problems in a systematic way.

The theories explored relied on—indeed, were virtually fully specified by—a rich variety of symmetries pioneered and articulated by Murray Gell-Mann. The new developments made it possible to apply QFT to new particles and new interactions and fully explain their structure.

In the following decades, QFT was extended to well-describe not only the electromagnetic force but also weak and strong interaction so that new Lagrangians were found which contain new classes of particles or quantum fields. The search still continues for a more comprehensive theory of matter and energy, a *unified theory of all interactions*.

Yoichiro Nambu (1921–2015), co-discoverer of field theoretic spontaneous symmetry breaking.

The new focus on symmetry led to the triumph of non-Abelian gauge theories (the development of such theories was pioneered in 1954 with the work of Yang and Mills) and spontaneous symmetry breaking (by Yoichiro Nambu). Today, there are reliable theories of the strong, weak, and electromagnetic interactions of elementary particles which have an analogous structure to QED: They are the dominant framework of particle physics.

A combined renormalizable theory associated with the gauge group SU(3) × SU(2) × U(1) is dubbed the *standard model of elementary particle physics* (even though it is a full theory, and not just a model) and was assembled by Sheldon Glashow, Steven Weinberg and Abdul Salam in 1968, and Frank Wilczek, David Gross and David Politzer in 1973, on the basis of conceptual breakthroughs by Peter Higgs, François Englert, Robert Brout, Martin Veltman, and Gerard 't Hooft.

Gerard 't Hooft (b.1946) proved gauge field theories are renormalizable.

According to the standard model, there are, on the one hand, six types of leptons (e.g. the electron and its neutrino) and six types of quarks, where the members of both groups are all fermions with spin 1/2. On the other hand, there are spin 1 particles (thus bosons) that mediate the interaction between elementary particles and the fundamental forces, namely the photon for electromagnetic interaction, two W and one Z-boson for weak interaction, and the gluons for strong interaction. The linchpin of the symmetry breaking mechanism of the theory is the spin 0 Higgs boson, discovered 40 years after its prediction.

**Renormalization group**

*Main article: History of renormalization group theory*

Parallel breakthroughs in the understanding of phase transitions in condensed matter physics led to novel insights based on the renormalization group. They emerged in the work of Leo Kadanoff (1966) and Michael Fisher (1973), which underlay the seminal reformulation of quantum field theory by Ken Wilson in 1975. This reformulation provided insights into the evolution of effective field theories with scale, which classified all field theories, renormalizable or not (cf. subsequent section). The remarkable conclusion is that, in general, most observables are *"irrelevant"*, i.e., the macroscopic physics is *dominated by only a few observables* in most systems.

During the same period, Kadanoff (1969) introduced an operator algebra formalism for the two-dimensional Ising model, a widely studied mathematical model of ferromagnetism in statistical physics. This development suggested that quantum field theory describes its scaling limit. Later, there developed the idea that a finite number of generating operators could represent all the correlation functions of the Ising model.

**Conformal field theory**

The existence of a much stronger symmetry for the scaling limit of two-dimensional critical systems was suggested by Alexander Belavin, Alexander Polyakov and Alexander Zamolodchikov in 1984, which eventually led to the development of conformal field theory, a special case of quantum field theory, which is presently utilized in different areas of particle physics and condensed matter physics.

**Historiography**

The first chapter in Weinberg (2005) is a very good short description of the earlier history of QFT. Detailed accounts of the historical development of QFT can be found, e.g., in Darrigol 1986, Schweber (1994) and Cao 1997a. Various historical and conceptual studies of the standard model are gathered in Hoddeson et al. 1997 and of renormalization theory in Brown 1993.

Varieties of approaches

Most theories in standard particle physics are formulated as *relativistic quantum field theories*, such as QED, QCD, and the Standard Model. QED, the quantum field-theoretic description of the electromagnetic field, approximately reproduces Maxwell's theory of electrodynamics in the low-energy limit, with small non-linear corrections to the Maxwell equations required due to virtual electron–positron pairs.

**Perturbative and non-perturbative approaches**

In the perturbative approach to quantum field theory, the full field interaction terms are approximated as a perturbative expansion in the number of particles involved. Each term in the expansion can be thought of as forces between particles being mediated by other particles. In QED, the electromagnetic force between two electrons is caused by an exchange of photons. Similarly, intermediate vector bosons mediate the weak force and gluons mediate the strong force in QCD. The notion of a force-mediating particle comes from perturbation theory, and does not make sense in the context of non-perturbative approaches to QFT, such as with bound states.

**QFT and gravity**

There is currently no complete quantum theory of the remaining fundamental force, gravity. Many of the proposed theories to describe gravity as a QFT postulate the existence of a graviton particle that mediates the gravitational force. Presumably, the as yet unknown correct quantum field-theoretic treatment of the gravitational field will behave like Einstein's general theory of relativity in the low-energy limit. Quantum field theory of the fundamental forces itself has been postulated to be the low-energy effective field theory limit of a more fundamental theory such as superstring theory.

Definition

Quantum electrodynamics (QED) has one electron field and one photon field; quantum chromodynamics (QCD) has one field for each type of quark; and, in condensed matter, there is an atomic displacement field that gives rise to phonon particles. Edward Witten describes QFT as "by far" the most difficult theory in modern physics – "so difficult that nobody fully believed it for 25 years."

**Dynamics**

*See also: Relativistic dynamics*

Ordinary quantum mechanical systems have a fixed number of particles, with each particle having a finite number of degrees of freedom. In contrast, the excited states of a quantum field can represent any number of particles. This makes quantum field theories especially useful for describing systems where the particle count/number may change over time, a crucial feature of relativistic dynamics. A QFT is thus an organized infinite array of oscillators.

**States**

QFT interaction terms are similar in spirit to those between charges with electric and magnetic fields in Maxwell's equations. However, unlike the classical fields of Maxwell's theory, fields in QFT generally exist in quantum superpositions of states and are subject to the laws of quantum mechanics.

Because the fields are continuous quantities over space, there exist excited states with arbitrarily large numbers of particles in them, providing QFT systems with effectively an infinite number of degrees of freedom. Infinite degrees of freedom can easily lead to divergences of calculated quantities (e.g., the quantities become infinite). Techniques such as renormalization of QFT parameters or discretization of spacetime, as in lattice QCD, are often used to avoid such infinities so as to yield physically plausible results.

**Fields and radiation**

The gravitational field and the electromagnetic field are the only two fundamental fields in nature that have infinite range and a corresponding classical low-energy limit, which greatly diminishes and hides their "particle-like" excitations. Albert Einstein in 1905, attributed "particle-like" and discrete exchanges of momenta and energy, characteristic of "field quanta", to the electromagnetic field. Originally, his principal motivation was to explain the thermodynamics of radiation. Although the photoelectric effect and Compton scattering strongly suggest the existence of the photon, it might alternatively be explained by a mere quantization of emission; more definitive evidence of the quantum nature of radiation is now taken up into modern quantum optics as in the antibunching effect.

Principles

**Classical and quantum fields**

*Main article: Classical field theory*

A classical field is a function defined over some region of space and time. Two physical phenomena which are described by classical fields are Newtonian gravitation, described by Newtonian gravitational field **g**(**x**,*t*), and classical electromagnetism, described by the electric and magnetic fields **E**(**x**, *t*) and **B**(**x**, *t*). Because such fields can in principle take on distinct values at each point in space, they are said to have infinite degrees of freedom.

Classical field theory does not, however, account for the quantum-mechanical aspects of such physical phenomena. For instance, it is known from quantum mechanics that certain aspects of electromagnetism involve discrete particles—photons—rather than continuous fields. The business of *quantum* field theory is to write down a field that is, like a classical field, a function defined over space and time, but which also accommodates the observations of quantum mechanics. This is a *quantum field*.

To write down such a quantum field, one promotes the infinity of classical oscillators representing the modes of the classical fields to quantum harmonic oscillators. They thus become operator-valued functions (actually, distributions). (In its most general formulation, quantum mechanics is a theory of abstract operators (observables) acting on an abstract state space (Hilbert space), where the observables represent physically observable quantities and the state space represents the possible states of the system under study. For instance, the fundamental observables associated with the motion of a single quantum mechanical particle are the position and momentum operators {\displaystyle {\hat {x}}} and{\displaystyle {\hat {p}}}. Field theory, by sharp contrast, treats *x* as a label, an index of the field rather than as an operator.)

There are two common ways of handling a quantum field: canonical quantization and the path integral formalism. The latter of these is pursued in this article.

**Lagrangian formalism**

Quantum field theory relies on the Lagrangian formalism from classical field theory. This formalism is analogous to the Lagrangian formalism used in classical mechanics to solve for the motion of a particle under the influence of a field. In classical field theory, one writes down a Lagrangian density, involving a field, φ(**x**,*t*), and possibly its first derivatives (∂φ/∂*t* and ∇φ), and then applies a field-theoretic form of the Euler–Lagrange equation. Writing coordinates (*t*, **x**) = (*x*0, *x*1, *x*2, *x*3) = *x*μ, this form of the Euler–Lagrange equation is{\displaystyle {\frac {\partial }{\partial x^{\mu }}}\left[{\frac {\partial {\mathcal {L}}}{\partial (\partial \varphi /\partial x^{\mu })}}\right]-{\frac {\partial {\mathcal {L}}}{\partial \varphi }}=0,} where a sum over μ is performed according to the rules of Einstein notation.

By solving this equation, one arrives at the "equations of motion" of the field. For example, if one begins with the Lagrangian density and then applies the Euler–Lagrange equation, one obtains the equation of motion:

This equation is Newton's law of universal gravitation, expressed in differential form in terms of the gravitational potential φ(*t*, **x**) and the mass density ρ(*t*, **x**). Despite the nomenclature, the "field" under study is the gravitational potential, φ, rather than the gravitational field, **g**. Similarly, when classical field theory is used to study electromagnetism, the "field" of interest is the electromagnetic four-potential (*V*/*c*, **A**), rather than the electric and magnetic fields **E** and **B**.

Quantum field theory uses this same Lagrangian procedure to determine the equations of motion for quantum fields. These equations of motion are then supplemented by commutation relations derived from the canonical quantization procedure described below, thereby incorporating quantum mechanical effects into the behavior of the field.

**Single- and many-particle quantum mechanics**

*Main articles: Quantum mechanics and First quantization*

In non-relativistic quantum mechanics, a particle (such as an electron or proton) is described by a complex wavefunction, *ψ*(*x*, *t*), whose time-evolution is governed by the Schrödinger equation:

Here *m* is the particle's mass and *V*(*x*) is the applied potential. Physical information about the behavior of the particle is extracted from the wavefunction by constructing expected values for various quantities; for example, the expected value of the particle's position is given by integrating *ψ*\*(*x*) *x* *ψ*(*x*) over all space, and the expected value of the particle's momentum is found by integrating −*iħ*ψ\*(*x*)d*ψ*/d*x*. The quantity *ψ*\*(*x*)*ψ*(*x*) is itself in the Copenhagen interpretation of quantum mechanics interpreted as a probability density function. This treatment of quantum mechanics, where a particle's wavefunction evolves against a classical background potential *V*(*x*), is sometimes called *first quantization*.

This description of quantum mechanics can be extended to describe the behavior of multiple particles, so long as the number and the type of particles remain fixed. The particles are described by a wave function *ψ*(*x*1, *x*2, …, *xN*, *t*), which is governed by an extended version of the Schrödinger equation.

Often one is interested in the case where *N* particles are all of the same type (for example, the 18 electrons orbiting a neutral argon nucleus). As described in the article on identical particles, this implies that the state of the entire system must be either symmetric (bosons) or antisymmetric (fermions) when the coordinates of its constituent particles are exchanged. This is achieved by using a Slater determinant as the wavefunction of a fermionic system (and a Slater permanent for a bosonic system), which is equivalent to an element of the symmetric or antisymmetric subspace of a tensor product.

For example, the general quantum state of a system of *N* bosons is written as where {\displaystyle |\phi \_{i}\rangle }are the single-particle states, *Nj* is the number of particles occupying state *j*, and the sum is taken over all possible permutations *p* acting on *N* elements. In general, this is a sum of *N*! (*N* factorial) distinct terms{\displaystyle {\sqrt {\frac {\prod \_{j}N\_{j}!}{N!}}}} is a normalizing factor.

There are several shortcomings to the above description of quantum mechanics, which are addressed by quantum field theory. First, it is unclear how to extend quantum mechanics to include the effects of special relativity. Attempted replacements for the Schrödinger equation, such as the Klein–Gordon equation or the Dirac equation, have many unsatisfactory qualities; for instance, they possess energy eigenvalues that extend to –∞, so that there seems to be no easy definition of a ground state. It turns out that such inconsistencies arise from relativistic wavefunctions not having a well-defined probabilistic interpretation in position space, as probability conservation is not a relativistically covariant concept. The second shortcoming, related to the first, is that in quantum mechanics there is no mechanism to describe particle creation and annihilation; this is crucial for describing phenomena such as pair production, which result from the conversion between mass and energy according to the relativistic relation *E* = *mc*2.

**Second quantization**

*Main article: Second quantization*

In this section, we will describe a method for constructing a quantum field theory called second quantization. This basically involves choosing a way to index the quantum mechanical degrees of freedom in the space of multiple identical-particle states. It is based on the Hamiltonian formulation of quantum mechanics.

Several other approaches exist, such as the Feynman path integral, which uses a Lagrangian formulation. For an overview of some of these approaches, see the article on quantization.